Human Capital Use, Innovative Activity, and Patent Protection in a Model of Regional Economic Growth

by

Amitrajeet A. Batabyal

and

Peter Nijkamp

---

1
We thank participants in an International Workshop in the Tinbergen Institute, Amsterdam, The Netherlands, and seminar participants at the Delhi School of Economics, New Delhi, India, the Indian Statistical Institute, New Delhi, India, and the Indian Institute of Technology, Mumbai, India, for their helpful comments on a previous version of this paper. In addition, Batabyal acknowledges financial support from the Gosnell endowment at RIT. The usual disclaimer applies.

2
Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. Internet aabgsh@rit.edu

3
Department of Spatial Economics, Free University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Internet pnijkamp@feweb.vu.nl
Human Capital Use, Innovative Activity, and Patent Protection in a Model of Regional Economic Growth

Abstract

We provide the first theoretical analysis of the aggregate effects of the trinity of human capital use, innovative activity, and patent protection, on regional economic growth. In our model, consumers have constant relative risk aversion preferences, there is no human capital growth, and there are three kinds of manufacturing activities involving the production of blueprints for inputs or machines, the inputs or machines themselves, and a single final good for consumption. Our analysis generates four salient results. First, we delineate the balanced growth path (BGP) equilibrium and show that the BGP growth rate depends negatively on the rate $\lambda$ at which patents expire. Second, we characterize the transitional dynamics in our regional growth model. Third, we determine the value of the patent expiry rate $\lambda$ that maximizes the equilibrium growth rate of the regional economy under study. Finally, we show that a policy of setting $\lambda=0$ (offering perpetual patent protection) does not necessarily maximize social welfare in our regional economy at time $t=0$.

Keywords: Dynamic, Human Capital, Innovation, Patent, Regional Economy, Stochastic

JEL Codes: R11, O31, L52
1. Introduction

Modern growth theory has rightly emphasized the endogeneity of economic growth in relation to entrepreneurship, political/economic leadership, creative classes, and the notion of social capital. In this context, education and skills are often considered to be critical factors by researchers. In fact, the last three decades have given rise to a great deal of discussion about the role of human capital in promoting the economic growth of regions. As noted by Faggian and McCann (2009), this is because there is increasing recognition that knowledge is the key driver of growth in most modern regional economies and that highly skilled workers are the key providers of this knowledge. Put a little differently, the available human capital in a region directly affects the production of final consumption goods. In addition, this human capital also indirectly influences the production of these final goods by engaging in R&D—which results in the invention of blueprints for new inputs—and by producing these new inputs.4

In addition to the salient role played by human capital in promoting the economic growth of regions, in recent times, researchers such as Fischer and Nijkamp (2009), Baumol (2010), and Batabyal and Nijkamp (2011) have also stressed the powerful role of innovative activities in enhancing regional growth and development. In the words of Fischer and Nijkamp (2009, p. 185), “entrepreneurial innovation is the essence of capitalism and its process of creative destruction, embodied in new products, new production processes and new forms of organisation.” A central conclusion emanating from this line of research is that regional growth and development are very closely related to the activities of innovative entrepreneurs.

Contemporary research on human capital in regional science has shed light on the many

4 In the remainder of this paper, we shall frequently refer to these inputs as machines.
nexuses between human capital use and regional economic growth. Florida et al. (2008) discuss educational and occupational measures of human capital and show that the occupational measure outperforms the educational measure when one attempts to account for regional labor productivity measured in wages. In contrast, the educational measure is the superior measure if one’s aim is to account for regional income. Using German data, Brunow and Hirte (2009) show that there are age specific human capital effects and that a temporary increase in regional productivity can occur during what these authors call the demographic transition. Fleisher et al. (2010) point out that investments in human capital in the “less-developed areas” of China are justified because they promote efficiency and contribute to the reduction of regional inequalities. Hammond and Thompson (2010) analyze trends in human capital accumulation in the United States and find scant evidence of convergence in college attainment across metropolitan and non-metropolitan areas. To address the problem of widening regional disparities in Slovakia, Banerjee and Jarmuzek (2010) suggest that policy focus on the ways in which human capital accumulation might be increased.

The largely empirical and case study based literature on innovative activity and patent protection in regional economies has addressed a number of pertinent issues. For instance, Acs et al. (2002) empirically examine the usefulness and the limitations of using patent and innovation counts to explain the regional production of new knowledge. Bottazzi and Peri (2003) use European patent data and contend that although a doubling of R&D spending in a region will substantially increase innovation in this region, this doubling will only have a minuscule impact on innovation in neighboring regions. In their study of differential patenting in Spanish regions, Gumbau-Albert and Maudos (2009) show that as expected, a region’s own R&D activities have a significant positive effect on innovation output measured by the number of patents. However, unlike the paper by
Bottazzi and Peri (2003), these two authors note that R&D spillovers are salient and that such spillovers result in positive effects on a region’s patents. Finally, Chapple et al. (2011) use surveys to make the point that although firms engaging in “green innovation” are likely to respond positively to local and regional markets, this kind of innovation does not necessarily foster regional growth.

The studies discussed in the preceding two paragraphs have certainly enhanced our understanding of the myriad connections between the trinity of human capital use, innovative activity, and patent protection, and the growth process in regional economies. This notwithstanding, to the best of our knowledge, there are virtually no theoretical studies that are both dynamic and stochastic in nature and that study the effects of the threesome of human capital use, innovative activity, and patent protection, on endogenous economic growth in a regional economy.

Given this state of affairs, in our paper, we analyze a dynamic and stochastic model of a regional economy in which consumers have constant relative risk aversion preferences, there is no human capital growth, and there are three kinds of manufacturing activities involving the production of blueprints for inputs or machines, the inputs or machines themselves, and a single final good for consumption. In this setting, we first delineate the balanced growth path (BGP) equilibrium and show that the BGP growth rate depends negatively on the rate $\lambda$ at which patents expire. Second, we characterize the transitional dynamics in our model. Third, we ascertain the value of the patent expiry rate $\lambda$ that maximizes the equilibrium growth rate of our regional economy. Finally, we show that a policy of setting $\lambda=0$ (offering perpetual patent protection) does not necessarily maximize social welfare in our regional economy at time $t=0$.

The rest of this paper is organized as follows. Section 2 describes our theoretical model of a regional economy which is adapted from the prior work of Rivera-Batiz and Romer (1991) and
Acemoglu (2009, pp. 433-444). Section 3 first delineates the BGP equilibrium and then studies the nature of the BGP growth rate’s dependence on the rate $\lambda$ at which patents expire. Section 4 discusses the conditions under which there are transitional dynamics in our model. Section 5 determines the value of the patent expiry rate $\lambda$ that maximizes the equilibrium growth rate of our regional economy. Section 6 demonstrates that a policy of setting $\lambda = 0$ (offering perpetual patent protection) does not necessarily maximize social welfare in our regional economy at time $t=0$. Finally, section 7 concludes and then discusses potential extensions of the research delineated in this paper.

2. The Theoretical Framework

2.1. Preliminaries

We begin by focusing on a stylized, infinite horizon regional economy in which there is human capital use, innovative activity, and patent protection. Consumers in our regional economy display constant relative risk aversion (CRRA) and their CRRA utility function in time $t$ is $\{C(t)^{1-\theta} - 1\}/(1-\theta)$, $\theta \neq 1$, where $C(t)$ is consumption in time $t$, and $\theta \geq 0$ is the constant coefficient of relative risk aversion.\(^5\) In what follows, we shall abstract away from issues related to the growth of human capital in our regional economy. In addition, we shall suppose that the total available human capital is supplied inelastically and that it does not respond to signals from the underlying economic system.

The single final good for consumption is produced competitively with the production function

\(^5\) See Mas-Colell et al. (1995, p. 194) for more on the properties of CRRA utility functions.
Since we are working with a model of endogenous technology, firms and individuals in our regional economy must ultimately have a choice between different kinds of technologies and, in this regard, greater effort, investment, or R&D spending ought to lead to the invention of better technologies. These features tell us that there must exist a meta production function or a “production function over production functions” which tells us how new technologies are generated as a function of various inputs. Following Acemoglu (2009, p. 413), we refer to this meta production function as the “innovation possibilities frontier.”

\[
Y(t) = \frac{1}{1-\beta} \left[ \int_0^{N(t)} x(v,t)^{1-\beta} dv \right] H^\beta, \tag{1}
\]

where \( H \) is the aggregate human capital input, \( N(t) \) denotes the different number of the varieties of machines that are used to produce the final good at time \( t \), \( x(v,t) \) is the total amount of the machine of variety \( v \) that is used at time \( t \), and \( \beta \in (0,1) \) is a parameter of the production function. We shall assume that the \( x(.,.) \)'s depreciate fully after they have been used. This assumption is made for two reasons. First, with this assumption, we can think of these \( x(.,.) \)'s as generic inputs. Second, because these \( x(.,.) \)'s depreciate immediately, in our subsequent mathematical analysis, we will not have to work with additional state variables. Note that for a given number of varieties of machines or \( N(t) \), the production function in equation (1) exhibits constant returns to scale. In addition, we normalize the price of the final consumption good at all time points to equal unity. Our next task is to discuss how new machines in our regional economy are invented and then produced.

### 2.2. Machine invention, production, and patent protection

Once the blueprint for a particular variety of machine has been invented, one unit of this machine can be produced at marginal cost equal to \( \psi > 0 \) units of the final consumption good. The so called “innovation possibilities frontier”\(^6\) in our regional economy is given by

\[
\frac{dN(t)}{dt} = \tilde{N}(t) = \eta Z(t), \tag{2}
\]

---

\(^6\) Since we are working with a model of endogenous technology, firms and individuals in our regional economy must ultimately have a choice between different kinds of technologies and, in this regard, greater effort, investment, or R&D spending ought to lead to the invention of better technologies. These features tell us that there must exist a meta production function or a “production function over production functions” which tells us how new technologies are generated as a function of various inputs. Following Acemoglu (2009, p. 413), we refer to this meta production function as the “innovation possibilities frontier.”
where $\eta > 0$ is a flow parameter and $Z(t)$ is the total expenditure on R&D in our regional economy. Inspecting equation (2), it is straightforward to verify that increased expenditure on R&D leads to a more rapid invention of new machines.

We assume that there is free entry into R&D activities and this means that any individual or firm in our regional economy can spend one unit of the final consumption good at time $t$ to create a flow rate $\eta$ of new blueprints. A firm that discovers a blueprint for a new machine, i.e., invents a new machine, receives a patent on this machine variety which expires at the policy determined Poisson rate $\lambda \geq 0$. When $\lambda = 0$, this means that full patent protection is offered in the sense that patent protection will never be removed. In contrast, $\lambda \rightarrow \infty$ implies that patent protection is removed immediately. When a patent on a machine expires, this machine is produced competitively and is supplied to the producers of the final consumption good at marginal cost. The $\mathcal{N}(0)$ number of initial machine varieties is supplied by monopolists with the above described patent structure. Note that because many different firms in our regional economy are engaging in expenditures on R&D activities, there is no aggregate uncertainty in the innovation process and hence equation (2) holds deterministically in our model.

Given the above patent structure, a firm that invents a new machine of variety $\nu$ is the monopolistic supplier of this variety and, as such, at any time $t$, it sets a price $p^*(\nu,t)$ to maximize profits. Note that because of our assumption of full depreciation of machines, this price $p^*(\nu,t)$ can also be interpreted as the user cost for this machine. The demand for a machine of variety $\nu$—on which more below in the next section—is given by maximizing the net total profit of the final consumption good sector.

We denote the net present discounted value of owning the blueprint of a machine of variety $\nu$
by the differentiable in time function $\mathcal{V}(v,t)$. The assumed time differentiability of our value function means that we can write this function in the form of a Hamilton-Jacobi-Bellman (HJB) equation given by

$$r(t)\mathcal{V}(v,t) - \dot{\mathcal{V}}(v,t) = \pi(v,t),$$

(3)

where $\pi(v,t) = p^*(v,t)x(v,t) - \Psi x(v,t)$ is the profit of the monopolist producing the machine of variety $v$ at time $t$, $p^*(v,t)$ and $x(v,t)$ are the monopolists’ profit maximizing price and quantity choices, and $r(t)$ is the interest rate at time $t$. Finally, the resource constraint affecting our regional economy at any time $t$ is

$$C(t) + X(t) + Z(t) \leq Y(t),$$

(4)

where $C(t)$, $Z(t)$, $Y(t)$ have been explained previously and $X(t)$ is total spending or investment on machines.

The reader should note that in this paper, when we refer to a “regional economy,” we are referring to the economy of a locality that is smaller than that of a nation-state. The structure of our theoretical model and the assumptions we have made in this second section of the paper are based on and consistent with this interpretation. In this regard, we contend that our focus on a single final good sector, our abstracting away from human capital growth issues, and our disregard of financial matters in general and the nature of the portfolio holdings of consumers in particular are defensible in the case of a regional economy—in the sense mentioned above—but not necessarily so in the context of a national economy. With this background in place, the task for us now is to characterize the BGP equilibrium for our innovative regional economy. In the course of undertaking this

---

See theorem 7.10 and the related discussion in Acemoglu (2009, p. 244, pp. 435-436) for more on the technical details of this procedure.
exercise, we shall also analyze the nature of the BGP growth rate’s dependence on the Poisson rate $\lambda$ at which patents on newly invented machines in our economy expire.

3. The BGP Equilibrium

3.1. Preliminaries

Let $N_a(t)$ denote the set of machine varieties whose patents have not expired and let $N_e(t)$ denote the set of machine varieties whose patents have expired. Also, let $r(t)$ and $w(t)$ denote the interest rate and the wage paid to the human capital input at time $t$. Then, an equilibrium in our regional economy is a collection of time paths of aggregate resource allocations, the set of machine varieties whose patents have not and have expired, quantities, prices, the value function for each machine, interest rates, and wages for human capital such that the following six conditions are satisfied. First, consumers choose consumption optimally. Second, the evolution of patented machines is determined by free entry in R&D and the expiration of patents. Third, machine producers with patents set prices to maximize profits. Fourth, machines with expired patents are produced competitively. Fifth, the final consumption good is produced competitively. Sixth, all markets clear. In symbols, the collection of trajectories that is of interest is

\[
\begin{align*}
[Y(t), C(t), X(t), Z(t), N_a(t), N_e(t), \{p^s(v,t), x(v,t), V(v,t)\}_{v \in N_d(0)}, \\
{\{p^s(v,t), x(v,t)\}_{v \in N_d(0)}, r(t), w(t)}_{t=0}. 
\end{align*}
\]

3.2. Static equilibrium allocations

Let us now characterize the static equilibrium allocations in our regional economy for given values of $N_a(t)$ and $N_e(t)$. From the discussion in Acemoglu (2009, p. 435), we infer that the
demand for machines from the producers of the final consumption good is isoelastic and is given by

\[ x(v,t) = p^x(v,t)^{-1/\beta}H. \] (6)

Equation (6) tells us that the demand for machines depends only on the price (user cost) of the machine and on the equilibrium supply of human capital. This demand does not depend directly on the interest rate \( r(t) \), the wage paid to human capital \( w(t) \), and the total number of machine varieties \( N(t) \).

Machine producers with patents will set the monopoly price. Therefore, given the isoelastic demand in equation (6), we can tell that

\[ p^x(v,t) = \frac{\Psi}{1-\beta} \quad \text{and} \quad x(v,t) = \left( \frac{\Psi}{1-\beta} \right)^{-1/\beta}H, \quad \text{for} \quad v \in N_a(t). \] (7)

Given the price and quantity in equation (7), the monopolists’ per period profits are

\[ \pi(v,t) = \left( \frac{\Psi}{1-\beta} \right)^{-(1-\beta)\beta} \beta H. \] (8)

In contrast to the above line of reasoning, machines with expired patents are priced at marginal cost. This tells us that

\[ p^x(v,t) = \Psi \quad \text{and} \quad x(v,t) = \Psi^{-1/\beta}H \quad \text{for} \quad v \in N_e(t). \] (9)

Given equations (7)-(9), we can now state an expression for the total output of the final consumption good in our regional economy. That expression is

\[ Y(t) = \frac{1}{1-\beta}H \Psi^{-(1-\beta)\beta} \left[ N_a(t)(1-\beta)^{(1-\beta)\beta} + N_e(t) \right]. \] (10)
Similarly, we reason that the equilibrium wages paid to human capital in our regional economy are given by

\[ w(t) = \frac{\beta}{1 - \beta} \psi^{-\alpha(1-\beta)^\beta} [N_u(t)(1 - \beta)^{1-\beta} + N_e(t)]. \] (11)

Finally, we note that the aggregate expenditure on machines or investment in the economy under study is

\[ X(t) = H \psi^{-\alpha(1-\beta)^\beta} [N_u(t)(1 - \beta)^{1/\beta} + N_e(t)]. \] (12)

We now proceed to discuss the dynamic tradeoffs confronting our regional economy.

### 3.2. Dynamic tradeoffs

We begin by noting that the value function \( V(v,t) \) for machine producers with patents satisfies the HJB equation

\[ r(t)V(v,t) = \pi(v,t) + \dot{V}(v,t) - \lambda V(v,t), \] (13)

where, given the patent protection policy discussed in section 2.2, the last term on the right-hand-side (RHS) of equation (13) captures the fact that with a flow rate of \( \lambda \), the firm loses the patent and when this happens, its monopoly power declines to zero. We are interested in studying equilibria in which \( Z(t) \) or the expenditure on R&D is positive at all points in time \( t \). This interest combined with the fact that there is free entry into R&D pins down the value function and hence we get \( \eta V(v,t) = 1 \). We now use this last result and the expression for \( \pi(v,t) \) in equation (8) to solve equation (13). After several algebraic steps, we can show that the interest rate \( r(t) \) in our regional economy is constant for all time \( t \) and is given by
Maximization of the representative consumer’s utility function over an infinite horizon with \( \rho \) as the time discount rate gives us the Euler equation

\[
\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} (r(t) - \rho).
\] (15)

Combining equations (14) and (15), it is straightforward to deduce that the rate of growth of consumption in our regional economy or \( g \) is also constant and is given by

\[
g = \frac{1}{\theta} \left[ \left( \frac{\Psi}{1 - \beta} \right)^{\psi(1 - \beta)\beta} \eta \beta H - \lambda - \rho \right].
\] (16)

Since consumption in our regional economy grows at a constant rate, we infer that

\[
C(t) = C(0) \exp(gt).
\] (17)

Next, we focus on the dynamics of the sets of machine varieties with unexpired and with expired patents given by \( N_u(t) \) and \( N_e(t) \) respectively. Some thought tells us that in our model, the evolution over time of \( N_u(t) \) and \( N_e(t) \) is described by two differential equations. These equations are

\[
\dot{N}_u(t) = \eta Z(t) - \lambda N_u(t), \text{ with } N_u(0) \text{ given}
\] (18)

and

\[
\dot{N}_e(t) = \lambda N_u(t), \text{ with } N_e(0) \text{ given},
\] (19)

where the term \( \lambda N_u(t) \) in equations (18) and (19) captures the fact that the patent for each machine
expires at the flow rate of $\lambda$.

Now, using equations (10), (12), (17), and the market clearing condition for the final consumption good, we infer that the total expenditure on R&D at time $t$ or $Z(t)$ is

$$Z(t) = \frac{1}{1-\beta} \int H \psi^{-(1-\beta)\beta} [N_u(t)(1-\beta)^{\frac{1}{\beta}}(\frac{1}{1-\beta}-(1-\beta)) + \beta N_e(t)] - C(0) \exp(gt).$$  \hspace{1cm} (20)$$

Substituting this value of $Z(t)$ from equation (20) into equation (18) gives us a set of differential equations for the two variables $N_u(t)$ and $N_e(t)$ with two initial conditions $N_u(0)$ and $N_e(0)$, which can be solved for a given initial value of consumption $C(0)$.

The important point to note now is that among the various possible choices for $C(0)$, only one choice gives a stable solution for $N_u(t)$ and $N_e(t)$ in which $N_u$ and $N_e$ grow asymptotically at the rate $g$ which is also the constant rate of growth of consumption in our regional economy. In addition, this solution satisfies all the equilibrium requirements in our model. Therefore, the equilibrium under consideration is saddle path stable and it is uniquely characterized by the above mentioned two differential equations for $N_u$ and $N_e$.

### 3.3. The equilibrium path

The basic objective of this paper’s third section is to analyze the BGP equilibrium. To this end, let us conjecture that $N_u$ and $N_e$ grow at the rate $g$ which is also the constant rate of growth of consumption in our regional economy. Given the system of differential equations in equations (18) and (19), we infer that the BGP values of $N_u$ and $N_e$ must satisfy

---

8 The unstable solutions can be safely discarded because they either violate the transversality condition or the resource constraint in our model.
Also, from equation (18), we deduce that

\[ Z(t) = \frac{\dot{N}_u(t) + \lambda N_u(t)}{\eta} = N_u(t)(g + \lambda)/\eta, \]

where the second equality on the RHS of equation (22) follows from our conjecture that along a BGP, \( N_u(t) \) grows at the constant rate \( g \). Given equation (22), on our conjectured BGP, equation (20) can be rewritten as

\[ N_u(0)(\frac{g + \lambda}{\eta})\exp(gt) = \]

\[ \left\{ \frac{1}{1-\beta} H\Psi^{-\beta/(1-\beta)} [N_u(0)(1-\beta)^1/\beta \left\{ \frac{1}{1-\beta} -(1-\beta) \right\} + \beta N_u(0) \frac{\lambda}{g} - C(0) \} \right\}\exp(gt). \]

Now, canceling the exponential term \( \exp(gt) \) from both sides of equation (23) and then collecting the \( N_u(0) \) terms, we get an equation that describes the initial level of consumption \( C(0) \) in our regional economy. That equation is

\[ C(0) = N_u(0) \left\{ \frac{1}{1-\beta} H\Psi^{-\beta/(1-\beta)} [\left\{ \frac{1}{1-\beta} -(1-\beta) \right\} + \beta \frac{\lambda}{g} - \frac{g + \lambda}{\eta} \} \right\}. \]

We will now need two sets of restrictions on the parameters of our model so that the transversality condition is satisfied and the growth rate of consumption in the BGP equilibrium is positive. To this end, we first suppose that
The inequality in (25) ensures that the described path of consumption growth—see equation (16)—satisfies the transversality condition in our model. Second, inspecting equation (16) we see that the inequality

\[
\left(\frac{\psi}{1-\beta}\right)^{(1-\beta)/\beta} \beta H - \lambda > \rho
\]

must hold for there to be positive growth in the BGP equilibrium. The reader should note that positive consumption growth is also needed to verify our stipulation—discussed in section 3.2—that we are interested in studying a scenario in which investment on R&D or \(Z(t)\) is positive in equilibrium.

Our analysis thus far in this third section of the paper leads to three salient conclusions. First, when the parametric restrictions described in (25) and (26) are satisfied and the initial values of the two classes of machine varieties \(N_u(0)\) and \(N_e(0)\) satisfy equation (21), there exists a BGP equilibrium in which the aggregate variables \(N_u(t), N_e(t), C(t), Y(t),\) and \(w(t)\) all grow at the constant rate \(g\) given by equation (16). Note that the BGP equilibrium that we have been studying thus far exhibits endogenous technological progress. Specifically, firms engage in R&D and then invent new machines. These firms do so because their patents allow them to sell their machines profitably to the producers of the final consumption good. Put differently, the incentives for profit drive R&D and R&D drives economic growth in the region under study. Second, equation (16) tells
us that as in many models of endogenous technological progress,⁹ there is a *scale* effect in our model of regional economic growth. Specifically, this means that the larger is the human capital input \( H \), the greater is the growth rate \( g \) of the key aggregate variables such as consumption in our regional economy.

Finally, equation (16) clearly shows that the growth rate \( g \) is a *negative* function of the Poisson rate \( \lambda \) at which patents expire in our regional economy. This last result has noteworthy implications for both patent policy and innovative activity in the regional economy under study. We shall discuss these implications in greater detail in section 5. However, for the moment, we simply note the following. A policy that grants *perpetual* patent protection \( (\lambda = 0) \) to new inventions of machines has the greatest possible positive impact on regional economic growth. In contrast, bearing in mind the parametric restriction given in (26), a policy that offers patent protection to new machine inventions for a *finite* amount of time \( (\lambda > 0) \) acts as a drag on growth in our regional economy. This completes our discussion of the BGP equilibrium. We now discuss the conditions under which there are transitional dynamics in the model under study.

### 4. Transitional Dynamics

To comprehend whether there are transitional dynamics in our model of regional economic growth, let us consider two cases. The first case has already been discussed in detail in section 3.3 above. This is the case in which the parametric restrictions in (25) and (26) are satisfied and the initial values of the two classes of machine varieties \( N_u(0) \) and \( N_c(0) \) satisfy the condition given in equation (21). As discussed in section 3.3, in this case a BGP equilibrium exists and there are *no* transitional dynamics in the model under study.

---

⁹ Many of these models are discussed thoroughly in Acemoglu (2009) and in Aghion and Howitt (2009).
The second case corresponds to the situation in which the initial values of the two categories of machine varieties $N_u(0)$ and $N_e(0)$ do not satisfy the condition in equation (21). When this happens, there will be transitional dynamics in our regional economy of the following sort. Specifically, the $N_u(0)/N_e(0)$ ratio will converge monotonically to the $g/\lambda$ ratio and the aggregate variables in our economy, i.e., $N_u(t)$, $N_e(t)$, $C(t)$, $Y(t)$, and $w(t)$ will asymptotically grow at rate $g$. Our next task is to ascertain the specific value of the patent expiry rate $\lambda$ that maximizes the equilibrium growth rate of our regional economy.

5. Optimal Patent Expiry Rate

We have already shown that the BGP growth rate in our regional economy is given by equation (16). Inspecting this equation, it is straightforward to determine that the value of the patent expiry rate $\lambda$ that maximizes the growth rate $g$ is $\lambda^* = 0$. In other words, if an appropriate regulatory authority would like to maximize the growth rate in our region then he ought to set the Poisson patent expiry rate equal to zero and thereby provide perpetual patent protection to newly invented machines.

To understand this result, consider the effects of any case in which the pertinent regulatory authority provides patent protection for a finite period of time. In symbols, this means that he sets $\lambda > 0$. When patents expire faster, incentives for innovative activities are lower. In other words, firms’ expected profits are lower for a given interest rate. Therefore, to induce entry into the R&D sector of our regional economy, the interest rate will have to decline. With a lower interest rate, consumers in our regional economy will demand a flatter consumption profile and they will also reduce their savings. This will lead to lower investment in R&D (lower $Z(t)$), and this lower investment will lead to lower growth (lower $g$) in our regional economy.
In sum, purely from the standpoint of the maximization of the equilibrium growth rate of our regional economy, it is optimal to offer perpetual patent protection on newly invented machines. However, is such a patent policy also desirable from the standpoint of the maximization of social welfare in our regional economy? More concretely, does the policy of setting $\lambda = 0$ necessarily maximize social welfare in our regional economy at time $t=0$? We now proceed to answer this question.


We begin with a discussion of the nature of the static and dynamic distortions present in our regional economy. In this regard, note that there are static monopoly distortions which reduce the net output for a given number of machines $N(t)$. In addition, there are dynamic distortions stemming from the fact that relative to a monopolistic firm, the marginal value of a new machine is higher to a social planner. This is because of two reasons. First, the social planner accounts for the effect of new machines on both wages and profits whereas the monopolistic firms are only concerned with profits. Second, because the social planner avoids the monopoly distortions, this planner produces a higher net output for a given number of machines $N(t)$. Since the marginal value of a new machine is higher to the social planner, the growth rate in the socially planned regional economy exceeds that in the BGP equilibrium.

In light of the above observations, note that patents have two effects in our regional economy. On the one hand, increasing the patent expiry rate $\lambda$ also raises the rate at which the production of machines becomes competitive and this increases the static output of the final consumption good for a given number of machines. This can be seen easily by examining equation (10). We see that the coefficient $(1-\beta)^{(1-\beta)/\beta}<1$ multiplies the variable $N_\mu(t)$. This means that for a given number of
machine varieties \( N(t) = N_a(t) + N_e(t) \), total output of the final consumption good \( Y(t) \) is increasing in \( N_e(t) \). The effect of patents through this channel is welfare improving because this effect clearly mitigates some of the static monopoly distortions. On the other hand, as we noted in our discussion in section 5, increasing the patent expiry rate \( \lambda \) decreases the growth rate in our regional economy. As discussed in the preceding paragraph, this growth rate is less than optimal to begin with. Therefore, increasing \( \lambda \) reduces welfare in our regional economy through this second channel.

Depending on the preferences of consumers in our regional economy, the first or the second effect may dominate and hence the net impact of increasing the patent expiry rate \( \lambda \) may or may not be welfare improving. This ambiguous result notwithstanding, we can still shed light on this question in specific cases. For instance, consider the case in which consumers are relatively impatient (with a high discount rate \( \rho \) ) and their intertemporal substitution of consumption over time is relatively low (\( \theta \) is relatively high). In this case, it is more likely that the first effect will dominate the second effect and hence increasing the patent expiry rate \( \lambda \) will result in a welfare improvement in our regional economy. Note that in this case, consumers care relatively more about current consumption and they dislike a growing consumption profile. Therefore, consumers are likely to prefer the immediate benefits of a more competitive market to the delayed benefits of the monopolistic market.

To achieve efficiency in our regional economy at time \( t=0 \), an appropriate regulatory authority will need to reduce the distortions arising from the monopolistic markups but this authority will also need to provide sufficient surpluses to the monopolists so that they have the right incentives to engage in innovative activities. As our discussion of the two effects of patents in this section has shown, a policy of offering perpetual patent protection (setting \( \lambda = 0 \) ) maximizes the growth rate of our regional economy but it does not, in any way, deal with the extant monopoly distortions in this
economy. In addition, from the previous paragraph we know that there exist parametric configurations in which the first effect of patents dominates the second effect. Clearly, when this happens, setting $\lambda = 0$ will not maximize social welfare in the economy under study. So, our general conclusion is that a policy of offering perpetual patent protection does not necessarily maximize social welfare in our regional economy at time $t=0$.

7. Conclusions

In this paper, we used a dynamic and stochastic model to conduct, to the best of our knowledge for the first time, a theoretical analysis of the effects of the trinity of human capital use, innovative activity, and patent protection, on regional economic growth. We first delineated the BGP equilibrium and then showed that the BGP growth rate is a negative function of the rate $\lambda$ at which patents expire. Second, we provided conditions under which there are no transitional dynamics in our model and conditions under which such dynamics arise. Third, we showed that a policy of offering perpetual patent protection (setting $\lambda^* = 0$) maximized the equilibrium growth rate of our regional economy. Finally, we demonstrated that a policy of initially offering perpetual patent protection does not necessarily maximize social welfare in our regional economy.

The analysis in this paper can be extended in a number of directions. In this regard, note that when $\lambda = 0$, it should be possible to decentralize the social planner’s solution by providing a linear subsidy on a monopolist’s production of machines and by financing this subsidy with a lump-sum tax on consumers. It would be useful to determine whether this conjecture is true. In addition, it would also be instructive to explicitly introduce heterogeneity into the model and then study whether the above mentioned subsidy/tax scheme does or does not increase wealth inequality in a regional economy of the sort analyzed in this paper. Studies that incorporate these features of the problem
into the analysis will increase our understanding of the ways in which the trinity of human capital use, innovative activity, and patent protection influence the growth and development of dynamic and stochastic regional economies.
References


